Evaluation, and management of unstable rock slopes by 3-D laser scanning

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Abstract: Population growth frequently causes expansion of urban populations and associated transportation facilities onto formerly uninhabited slopes, where naturally occurring and constructed rock faces present potential hazards from rock fall and rock face instability.

Rock fall management and mitigation programs are widely used to ensure public safety, to provide efficient rock face maintenance, and to proactively address potential rock fall hazards. These programs require comprehensive geologic characterization of individual rock slopes. New ways of collecting data are urgently required to accelerate field data acquisition, reduce identification difficulties, eliminate safety constraints, and remove human bias and subjectivity in the data.

Exposed rock surfaces can be scanned quickly and accurately using a ground-based laser scanner, resulting in a collection of millions of discrete points each having a 3D value relative to the scanner's position. These "point clouds" have high spatial resolution, with adjacent points often only a centimeter or two apart. Relatively small surface features and discontinuities are readily identified. Algorithms have been developed to support semi-automated processes for orienting the point cloud, creating a polygonal surface model from the point cloud, determining rock discontinuity information, and defining block size distributions.

Résumé: La croissance de la population cause souvent l'expansion des populations urbaines et des équipements de transport associés sur des pentes autrefois inhabitées, dont les surfaces naturelles ou artificielles présentent un danger potentiel causé par la chute de roche ou par l’instabilité des parois de la roche.

Les programmes de gestion et de réduction de chute de roche sont largement employés pour assurer la sécurité publique, pour effectuer l'entretien efficace des parois de roche, et pour prévenir des risques potentiels de chute de roche. Ces programmes exigent la caractérisation géologique complète des différentes pentes de roche. De nouvelles méthodes de collecte de données sont exigées pour, accélérer l’acquisition de données sur le terrain, réduire les difficultés d'identification, éliminer les contraintes de sécurité, et réduire la subjectivité dans l’interprétation des informations.

Les surfaces exposées de la roche peuvent être balayées rapidement et d’une façon précise à l'aide d'un scanner à laser au sol, ayant pour résultat le relevé de millions de points discrets ayant chacun une position à trois dimensions relativement à la position du scanner. Ces « nuages de points » ont une résolution spatiale élevée, avec souvent des distances de un à deux centimètres entre deux points adjacents. Les détails relativement petits et les fissures du terrain sont aisément identifiés. Des algorithmes ont été développés et des procédés semi-automatisés ont été créés pour, orienter le nuage de points, créer un modèle polygonal surfacique à partir du nuage de points, déterminer les informations relatives aux fissures de roche, et définir la distribution des dimensions des blocs.

Keywords: fractures, highways, joints, remote sensing, site investigation, slope stability.

INTRODUCTION

In many parts of the world, spurred by population growth and demands for recreation and winter sports, urban populations and associated transportation facilities are expanding into mountainous regions. Formerly uninhabited slopes are now the locations of houses and transportation infrastructure, and naturally occurring and constructed rock faces present potential hazards from rock fall and rock face instability.

Rock fall management and mitigation programs are widely used to ensure public safety, to provide efficient rock face maintenance, and to proactively address potential rock fall hazards. These programs require comprehensive geologic characterization of individual rock slopes. New geotechnical/surveying technologies, including GPS, digital methods for field surveying and data collection, the use of still and video digital cameras, and the utilization of GIS and associated software for data processing and visualisation, are being applied to accelerate field data acquisition, reduce identification difficulties, eliminate safety constraints, and remove human bias and subjectivity in the data.

Assessment of rock slope instability requires the detailed information on the geometry of the exposed rock face and definition and analysis of discontinuity properties (joints, bedding planes, and fractures) of the rock mass because these determine, to a large extent, the mechanical behaviour (Bieniawski 1989). Ground-based 3D laser scanning represents an especially valuable new technology for providing detailed information on exposed rock faces (Slob, Hack & Turner 2002). The interest in using this new technology is largely driven by dissatisfaction with traditional manual field survey methods that have several disadvantages (Kemeny & Post 2003):

- Erroneous data are introduced due to sampling difficulties, e.g. choice of sampling method, human bias, instrument error, etc.
- Safety risks are often considerable. Often field measurements must be carried out at the base of existing slopes or during quarrying, tunneling, or mining operations, or along busy highways or railway tracks.
- Direct access to rock faces is often difficult or impossible.
- Apart from these practical problems, manual field survey methods are also time-consuming, labor-intensive and therefore costly.

In contrast, ground-based laser-scanning methods, often in association with digital images taken at the site, can be used to determine most of the required geotechnical information (Donovan et al. 2005a; Donovan et al. 2005b; Kemeny et al. 2004; Nasrallah et al. 2004; Slob et al. 2004; Slob et al. 2005). Laser scanning can reduce or eliminate access issues and safety concerns, and potentially reduce the time and costs associated with conventional site characterization methods.

PRINCIPLES OF 3D LASER SCANNING

The use of lasers to determine distances to objects is based on the same principles as ordinary radar; but "laser radar", or "Lidar," systems send out a narrow pulsed beam of light rather than broad radio waves. The systems utilize the speed of light and very precise timing devices to calculate the distance between a laser emitter/receiver device and an object reflecting the beam.

Laser ranging devices have been developed for use in mobile (airborne) platforms and are fairly widely used to develop accurate terrain models. Around 1998, a new class of laser scanning instruments was developed for use in ground-based near-range and highly accurate surveying applications. These instruments undertook a "progressive scan" of a desired scene by sending out a sequence of narrowly focussed laser beams with gradually changing orientations. Thus, provided the base unit was in a stable configuration during the scanning operation, a series of closely spaced but slightly offset objects would be illuminated by the laser and located by range and orientation to the instrument location. These devices were capable of generating dense "clouds of points" that could be processed to yield three-dimensional [x, y, z] definitions of the features being scanned. More importantly, these ground-based laser scanners could assess vertical, or near-vertical, rock faces that could not be accurately measured from the airborne sensors.

Technological and Economic Developments

The early devices were somewhat limited by current standards, but were the subject of considerable interest. Rapid improvements in the timing hardware allowed much more rapid sequencing of the pulses, and hence the more rapid collection of larger numbers of individual points. At the same time, improvements in laser technologies increased the focus of the beam and also reduced its tendency to enlarge (spread) with distance. Also the power of the laser, while maintaining visual safety issues, was adjusted so that maximum ranges of these instruments grew from a few hundred meters to over a kilometre. As these capabilities developed, in particular the longer maximum range, geologists and geotechnical engineers became increasingly interested in applying these instruments to rock slope assessments.

Table 1 summarizes the important characteristics of three ground-based laser scanners commonly used in North America. The data is mostly reproduced from technical literature provided by the manufacturers, although a few values (noted in the table) were estimated in order to assist the making of comparisons. These data reveal that current instruments are capable of collecting data at rates over 2000 points per second, with a position accuracy of about 5 mm at distances up to 800 meters. There are several trade-offs between accuracy, maximum distance and precision (Jacobs 2005a). The output from a typical laser-scanner survey is a "point cloud" consisting of millions of reflection points. These points will have 3D coordinates, plus a reflected laser intensity value. In addition, all manufactures now offer high-quality digital cameras, usually of 6 mega pixels or more, that are "bore sighted" with the laser scanner instrument. A technique called texture mapping or photo draping can be used to overlay this high-resolution colour information from digital camera images onto the 3D points, so each has additional R, G, B attributes.

These ground-based laser scanners have remained relatively expensive since their inception. A single scanner costs roughly ten-times the cost of a total station survey instrument. In spite of this cost difference, several state highway agencies have purchased ground-based laser scanners, often for assessing bridge structures, and geological personnel have experimented with the use of these scanners for rock slope assessments. A larger number of scanners have been purchased by private surveying firms, who have found their productivity increase over traditional approaches more than offsets their purchase cost (Jacobs 2005b). Experience by the authors demonstrates that these economic assessments are also valid for rock slope studies. For example, in 2003 a series of laser scans were used on a roadway widening project in Arizona and compared to traditional assessment methods that were also undertaken at the same locations. The total cost of the traditional methods was about $6250.00 (mostly manpower), while the laser scanning surveys cost about $5000.00 (one-third manpower, two-thirds equipment and software expenses). In addition, the traditional surveys required 10 days to complete, while the laser scanned products were available in two days. The conclusion that can be drawn from this comparative study is that laser scan-based survey and automated analysis can be considerably faster, less labour-intensive and therefore cheaper than traditional survey and analysis (Slob et al. 2005).

Table 1. Basic Characteristics of Popular 3D Laser Scanners
(Source: Manufacturer's literature and POBOLINE 2005)

Performance	Optech **ILRIS-3D**	Riegl **LMS-Z420i**	Leica **HDS3000**
Laser wavelength (nanometers)	1,500	1,550	532
Beam Diameter at 100m distance	27mm	25mm	12mm*
Average Data Acquisition Rate (pt/sec)	2,000	8,000	1,400
Maximum Data Acquisition Rate (pt/sec)	2,000	12,000	1,800
Distance Accuracy at 100m distance	7mm	5mm	8mm*
Position Accuracy at 100m distance	10mm	6mm	12mm*
Minimum Range	3m	2m	< 1m
Maximum Range [range limit depends on target albedo (0-100%)]	1500m @ 80% 700m @ 10%* 350m @ 4%	1000m @ 80% 350m @ 10%	400m @ 80%* 100m @ 5%
Digital Camera [externally mounted and boresighted]	6 megapixel	6.1 or 8.2 megapixel	6+ megapixel

* These values estimated from available published values.

Applications to Assessment of Rock Slopes

There are two important and distinctive applications of ground-based laser scanning for the investigation of rock slopes:

- Three-dimensional geometry information for volume and mass calculations. A laser-scan survey can be used to define the 3D geometry of a rock face. This information can be used in the design phase to accurately estimate the amount of material that must be scaled or excavated at a site. If a second survey is conducted after the excavation has been undertaken, a comparison of the two geometries can be used to estimate the amount of material that was actually removed and the volumes so computed may be used to determine pay quantities. Similarly, such before and after surveys can be used to determine the amount and location of rock falls, thus providing important information for proposing rock fall remediation.
- Geologic structure evaluation for geotechnical site investigation. Ground-based laser-scan surveys can be used to collect important geotechnical information, including fracture information (orientation, length, spacing, roughness, etc.), geological conditions, weathering, and hydrology.

These two applications have distinctive requirements for field operations and especially the establishment of survey control at the field site. The data produced by a laser scanner is referenced relative to the scanner's orientation and position. The scanner's position is defined as the origin: [0,0,0] and the scanner axis is the "vertical" axis. Such data have to be referenced to a regional or a local grid system if multiple scans are to be compared, or the data are to be used within existing databases in CAD or GIS.

Current laser-scanning systems provide software to allow real-time or posterior georeferencing of the point cloud data using reflectors that have pre-measured coordinates determined by a total station for example. The distribution and number of such control points and the accurate determination of their positions in the field and in the point cloud are especially critical for volume and mass calculation applications.

However, if the application is to determine rock-mass discontinuity properties, then the laser scanner data can be georeferenced to a local or regional grid or coordinate system with somewhat less precision. After all, traditional geological structure observations are made with hand-held compasses that have accuracies of about 1° at best. For example, slope stability analysis calculations rely on the relative orientation of joints and bedding planes compared to the actual slope orientation and geometry. The authors (Slob et al. 2005) have performed several successful geological structure analyses by placing two or more small (60x60 cm) white plywood boards with known orientations within the scanning field. Posterior processing of the scanned data allow transformation of the data to geographic orientations with accuracies comparable to traditional geological field surveys.

ANALYSIS OF POINT CLOUD DATA

Ground-based laser scanning produces very large point cloud data sets containing thousands to millions of 3D data points. These data are voluminous and often unlike any data sets familiar to geologists, geotechnical, or design engineers. Thus their subsequent usefulness and storage and management often requires new data handling software and training of personnel. Yet these point clouds are providing new opportunities for visualisation, as shown by the example in Figure 1.

Figure 2 summarizes the typical sequence of data processing steps. The three white boxes across the top define steps that are undertaken at the field site. They involve the basic conversion and transformation of the point cloud into 3D coordinates and the creation of primary data files. These are often quickly displayed and reviewed in the field to assure that adequate and correct data have been acquired.

Figure 1. A typical point cloud representation of a highway rock cut. Visible rock cut is about 10 meters high.

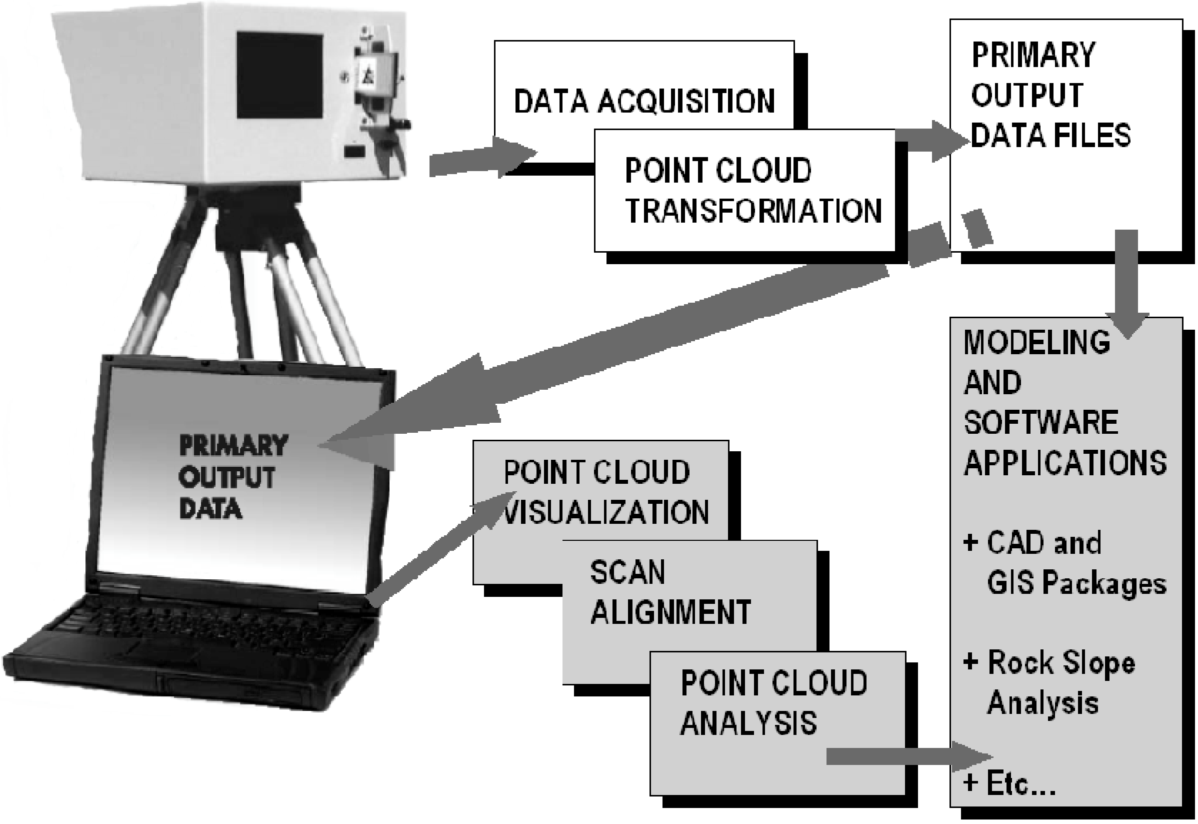

Figure 2. Typical sequence of data processing steps for laser scanning data collected to evaluate rock slopes.

The shaded boxes across the lower portion of Figure 2 identify the critical initial analysis steps. A number of commercial (proprietary) software packages are available from the laser scanner manufacturers to efficiently display the point clouds. A number of public-domain software products can also perform this task. It is often necessary to combine data from multiple scans because some features may be hidden or only partially visible from a single location. Therefore, scanner manufacturers generally provide software to support such operations, although the capabilities of the various packages are not uniform. As noted previously, depending on the application, greater or lesser attention must be paid to the absolute location and orientation of different scanned data sets.

The actual analysis of the point cloud data can be accomplished in combination with other “traditional” spatial software applications, including CAD and GIS. However, ground based laser scanners often collect spatial data with orientations that were not considered in the design of especially GIS, sometimes causing difficulties. For example, many popular GIS products do not support a full 3D spatial representation, but only a 2.5D data model that assumes the z-dimension can have only a single value and any given x – y location (Turner 2003). Some terrain modelling CAD systems also use 2.5D data models because they offer some computational efficiency and are acceptable for their expected use. Such systems cannot support the representation of vertical or overhanging features or elements, and so may be unacceptable in processing laser scans of vertical, or nearly vertical, rock faces.

Figure 2 also suggests that the analysis of point cloud data to evaluate rock slope conditions may be combined with traditional rock slope analysis software. In fact, the laser scanner manufacturers do not have the expertise, or the inclination, to satisfy a small, specialised application market. Traditional rock mechanics and structural geology applications software products have never expected to process the millions of points in a point cloud, and so often cannot accept the laser scanner data without some intermediate processing. Thus the broad acceptance of laser scanning will depend on the emergence of new data processing tools. Without them, the geologists and design engineers cannot efficiently use the point cloud data.

Surface reconstruction

Point cloud data provide a very good visual impression of the scanned object (see Figure 1). However, further analysis requires interpolation of the point cloud data to construct a true 3D surface model. 3D surface reconstruction algorithms can roughly be divided into Polygonal and Parametric. Delauney triangulation is a polygonal technique that creates irregular, triangular facets based on simple linear interpolation between the points in 3D space. As shown in Figure 3, the orientations of these individual triangular facets can be evaluated. Where groups of triangles have a relatively uniform orientation, they define a relatively planar surface, which may be an exposed rock joint or bedding plane.

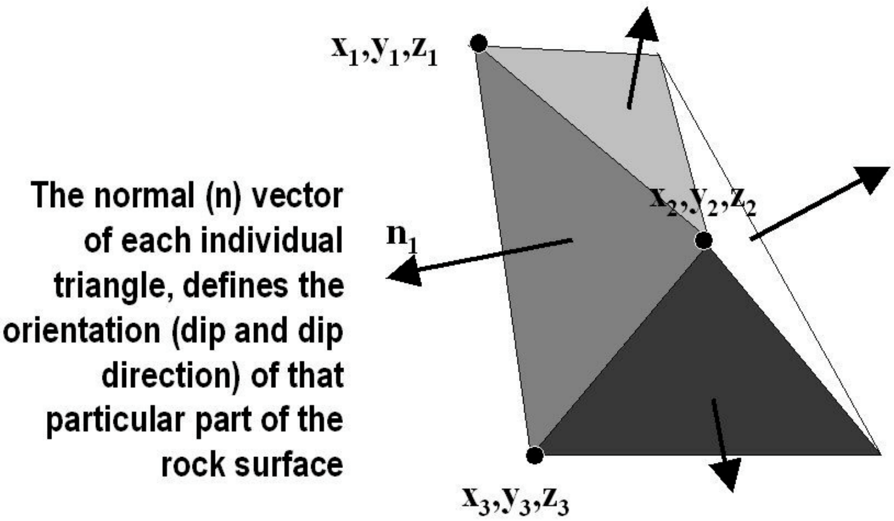

Figure 3. Orientation analysis of triangulated surface facets using normal vectors.

Delauney 3D triangulation techniques do not work well if the spatial density of the point cloud is relatively high compared to the laser range error. For instance, if a portion of a rock face is scanned with a spatial resolution of 5 mm, while the range error (more or less normal to the face) is in the order of 10 mm, then the interpolated triangular facets are oriented in an extremely noisy fashion. This problem can be partially overcome by under-sampling the point cloud data, but this effectively decreases the scanner resolution and details of the surface will be lost. An alternative is to use a local averaging approach to defining the corners of the triangles. This effectively smoothes the ranging error; the Split-Fx software developed by Split Engineering uses this approach (Split Engineering Home Page 2005).

Alternatives to resolving this problem involve the use of parametric techniques and applications, including NURBS (Non-Uniform Rational B-Splines) and Radial Basis Functions (RBF’s). Parametric interpolation using polyharmonic RBF’s allows construction of smooth, manifold surfaces from (noisy) point cloud data and can repair incomplete meshes (Carr et al. 2001; 2003). This technique allows fast reconstruction of surfaces, even on the basis of millions of points, something that was not possible before. The RBF produces, in effect, a solid model, from which gradients and surface normals can be determined analytically. This allows the user to create uniform meshes and this has advantages for mesh simplification and re-meshing applications (Carr et al. 2003).

Identification of Near-Planar Surfaces

Once the point clouds have been converted to surfaces, by any of the above methods, further analysis is possible. Figure 4 shows a small section of such a surface representing a section of a rock surface. Several large joint surfaces are readily apparent. The Split-Fx software created this mesh using data from a single scanning location. So some "holes" occur in the mesh where the surfaces were either not visible to the scanner, or were not adequately exposed. Also, a few noisy points have been included in the surface. An insect or bird (or vehicle or person) passing during the scanning process could cause such points; they are readily identified and removed form the data set. While such an interpolated surface provides considerably more information than the original point cloud, further information extraction is desired.

Figure 4. Detail of a triangulated mesh representing a small portion of a rock surface

It is of course possible to manually examine such a mesh and identify sections that appear to define specific rock mass discontinuities – rock joints, faults, or bedding planes – however, it is much more efficient to have the assistance of software in performing this task. Several methods are possible.

Figure 5 shows one result. The software has been asked to find groups of triangles having similar orientations – those falling within a defined tolerance. Subsequently the sizes of the patches, as defined by the numbers of adjacent triangles have been computed, and the largest such patches have been highlighted. It is clear from examination of Figure 5 that these define a dominant joint or bedding plane.

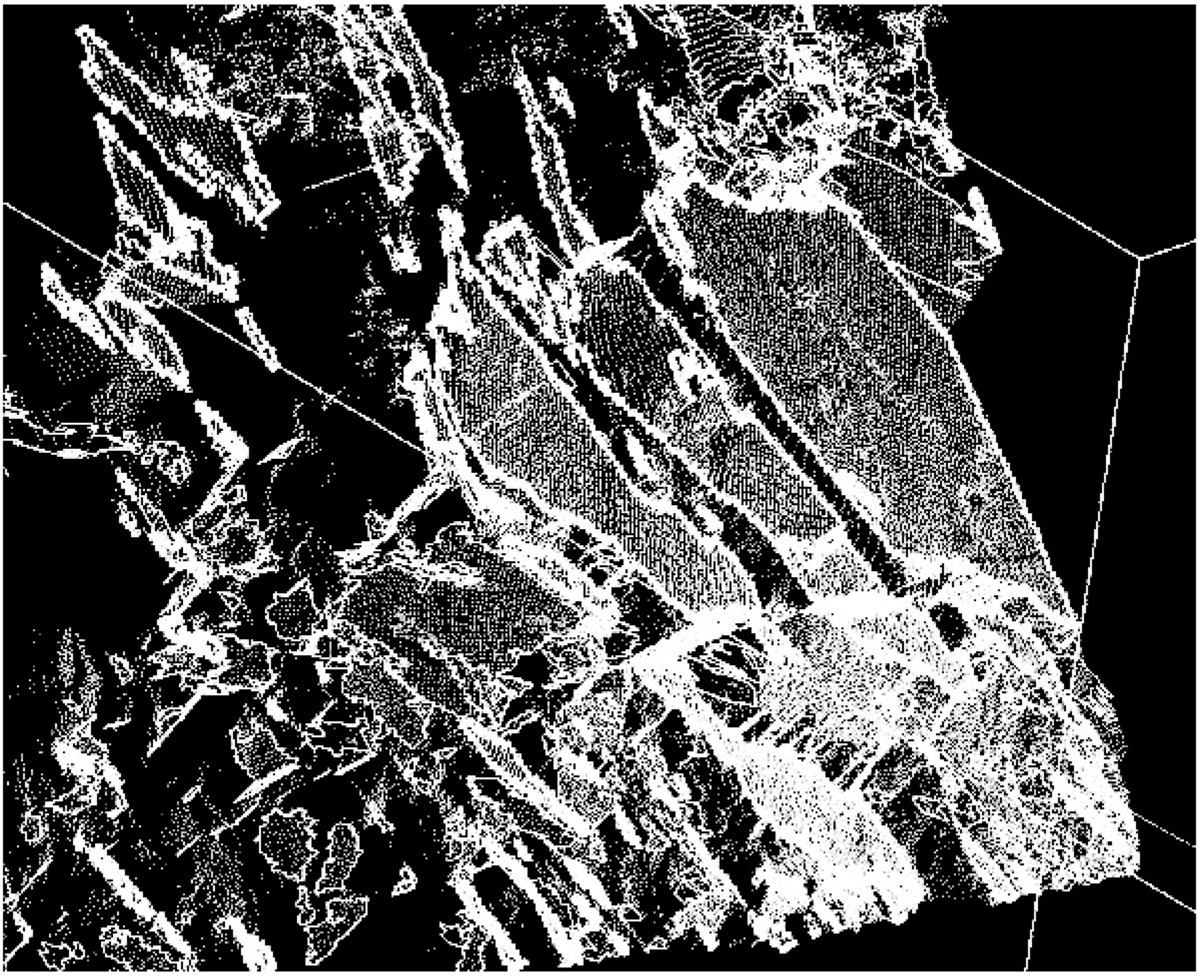

Figure 5. Interactive identification of a joint set or bedding plane

The example shown in Figures 4 and 5 illustrates the potential efficiencies of rock slope stability analysis with point cloud data obtained by laser scanning methods. However, the method will not be efficient if only manual manipulation of the point cloud data and the derived surfaces are used. Nor is a fully automated computer-driven methodology of analysis feasible or desirable.

What is desirable is an iterative process, in which the geologists and design engineers are given the opportunity to interrogate the 3D point cloud and the derived surfaces and progressively extract meaningful geological structures and rock mass discontinuities. Two approaches are under investigation – one approach utilizes a long-standing tool of the structural geologist, stereonets, while the second method combines the 3D laser scanning data with more classical image interpretation methods. These approaches are discussed in the following sections.

USE OF STEREONETS TO DEFINE ROCK MASS DISCONTINUITIES

By plotting the orientations of all individual facets in a stereo or polar plot, the trends in the data can be visualized and recognized in the form of clusters (Slob, Hack & Turner 2002; Slob et al 2004). Because of the high density of the laser data, it is possible for a single rock outcrop to have hundreds of thousands to millions of facets, providing a solid basis for statistic analyses.

Slob and his co-workers in ITC decided to apply automated clustering methods in order to eliminate human bias as much as possible. After some experimentation, the adjusted fuzzy k-means clustering method, suggested by Hammah and Curran (1998) was selected at the best approach. This method has the disadvantage that an initial guess of the number of clusters has to be made, which may introduce some human bias. However, the number of sets closest to reality can be determined with the aid of validity indices, as proposed by Gath and Geva (1989) and Xie and Beni (1991). One major advantage is that this algorithm can be improved through the use of quantifiable discontinuity properties, such as spacing, or joint roughness (Zhou & Maerz 2002).

Figures 6 and 7 show this process for a small example measured in Spain (Slob et al. 2005). Figure 6 shows the identified four major joint sets on a stereonet and on the 3D surface model without any restriction concerning uniformity. Figure 7 shows the same four joint sets where a validity index has been used to select only those closest to the mean orientations of each set.

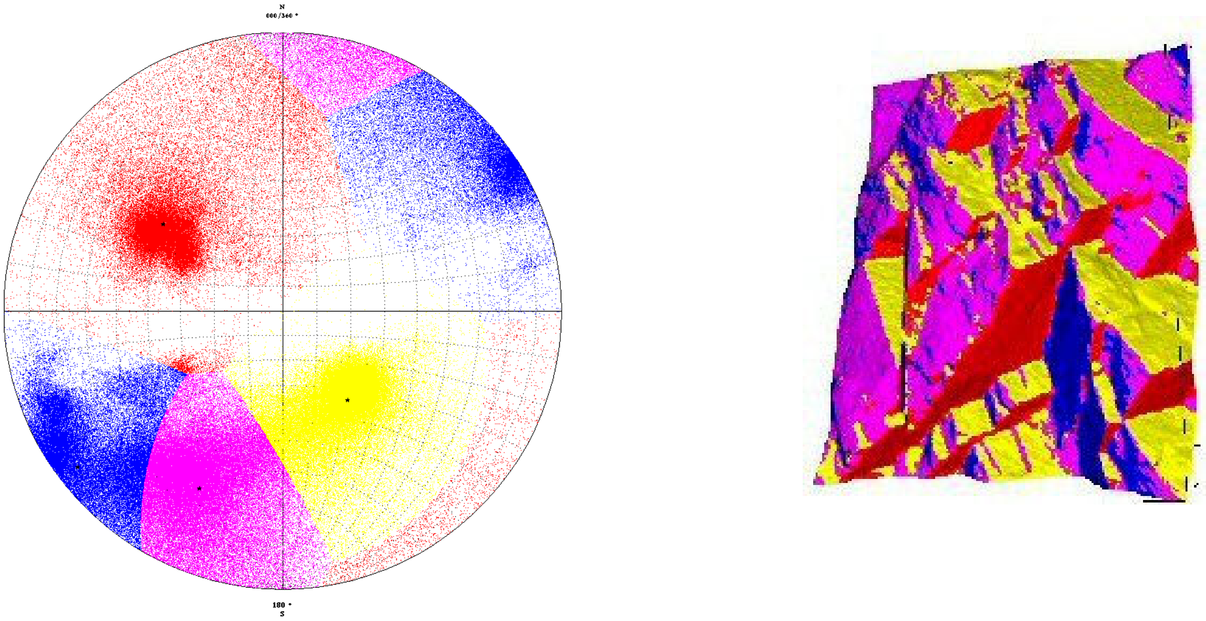

Figure 6. Four major joint sets defined using all facets shown on a stereonet and on the 3D surface model

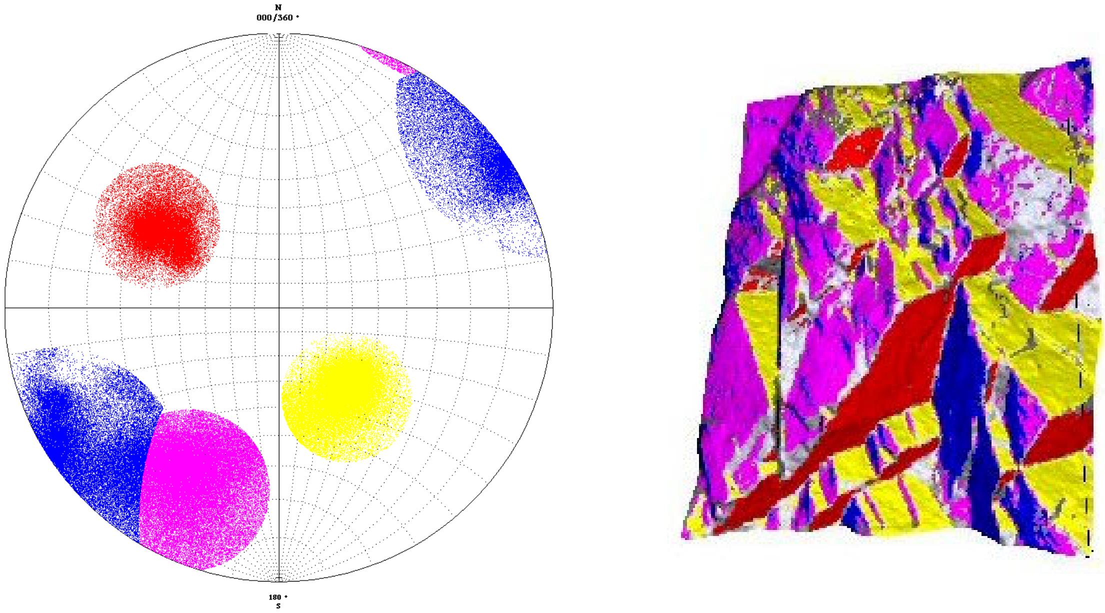

Figure 7. Four major joint sets defined with uniformity constraints shown on a stereonet and on the 3D surface model

COMBINING 3D LASER SCAN INFORMATION WITH 2D DIGITAL IMAGE ANALYSIS

In some cases rock discontinuity surfaces are readily apparent on rock outcrops. But this is not always true. Especially where joints or bedding planes are oriented more or less perpendicular to the rock face, they may not be exposed as surfaces, but yet are visible as lines or cracks in the rock face.

A good example of this phenomenon was encountered along a road in the mountains of south-western Colorado. The road follows the side of a deep gorge and at one point skirts a mountain ridge with an almost continuous rock face exposed. The rock is metamorphic gneiss. On one side the rock joints were clearly visible (Figure 8), while around the corner the rock face, oriented almost at right angles to the first exposure, exhibited a very different character. This second face is shown in Figure 9. The rock discontinuities so clearly exposed in Figure 8, are merely dark lines on the cliff face in Figure 9, where they rarely show any 3D offsets.

A relatively quick (15 minute duration) laser scan was conducted at the exposure shown in Figure 8 and yielded 1.5 million 3D points. Because of the irregularity of the rock face, it was not ideal for 2D digital imaging processing, but might be suitable for image analysis after the digital photo images were draped onto the 3D model surface. As might be expected, it was relatively easy to identify several dominant joint sets and foliations by analysing the laser point cloud data.

The situation shown in Figure 9 was just the opposite. The relatively smooth rock face yielded little discontinuity information from the 3D point cloud data. On the other hand, 2D image analysis techniques, especially edge detection algorithms, easily identified many prominent cracks in the rock face. The two exposures were an ideal location to combine the strength of the two methods.

The dominant structural discontinuities visible in the laser point cloud analysis at the Figure 8 exposure provided orientation statistics that could be used to make predictions of apparent dips of their exposures on the rock face exposure shown in Figure 9. These apparent dips were found to match in several cases with prominent fractures identified by the digital image analysis procedures. More importantly, it was possible to estimate how uniform or random the exposure of each defined discontinuity set might be on the rock face exposed at Figure 9. In this case, and in other similar situations, the combination of 3D laser scanning and 2D digital image analysis is able to produce a better and more complete structural analysis, than either method can on its own.

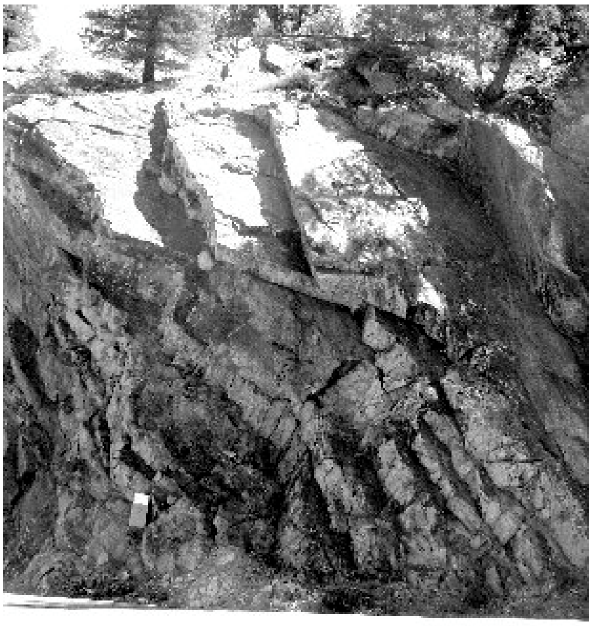

Figure 8. Colorado rock slope Photo 1. Height of view about 8 meters. At this exposure, several joints and foliations form prominent exposed surfaces.

Figure 9. Colorado rock slope Photo 2. This exposure is around the corner from Photo 1 and is approximately 90° (perpendicular) to that exposure. Here the prominent large exposed surfaces visible in Figure 8 show on the image as dark lines that can be easily found with edge detection algorithms.

FUTURE RESEARCH DIRECTIONS

There is a need to continue to evaluate 3D laser scanner developments and data analysis procedures, to conduct instrument comparisons at field-test facilities with well-documented rock slopes, and to verify laser data collection procedures. Several transportation agencies in the United States, as well as several geological surveys, have begun to apply ground-based laser scanning to the evaluation of rock slopes. Several have purchased or have access to scanners by various manufacturers. Yet there is no clear definition of the differences among these scanners, what are appropriate field procedures, and what are the trade-offs among different approaches.

These field evaluations should include comparisons of 3D laser scan data acquired for the same rock mass at different resolutions and scales, distances between the instrument and the rock face, etc. Additional experiments should be undertaken to compare ground and airborne laser scanning data to determine the scale effects. It is also important to further study the advantages of integrating laser scan data with the interpretation of optical imagery, especially the high resolution digital photographs now available with several scanners.

It appears important to investigate whether the intensity of the reflected laser returns, when combined with information on the orientation of the local rock surface, provides any information that might assess the roughness and weathering of individual discontinuity sets.

Laser scanning of exposed rock faces appears to ultimately offer an efficient and rapid method of obtaining probabilistic distributions of block sizes within the rock mass. Therefore future research will be oriented toward developing software routines to:

- Statistically recognize discontinuity patterns in the 3D data using spherical data distribution analysis and determine variance within each discontinuity data cluster.
- Calculate discontinuity spacing distributions for each identified discontinuity set from the 3D TIN surface model.

Then it should be possible to develop probabilistic block-size distributions from the above calculations. Such probabilistic block size distributions could form the basis for the design of appropriate catchment devices, or to evaluate rock fall hazard ratings.

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REFERENCES

BIENIAWSKI, Z.T. 1989. *Engineering Rock Mass Classification*. Wiley, New York, USA, 251p.

CARR, J.C., BEATSON, R.K., CHERRIE, J.B., MITCHELL, T.J., FRIGHT, W.R., MCCALLUM, B.C. & EVANS, T.R. 2001. Reconstruction and Representation of 3D Objects with Radial Basis Functions. *ACM SIGGRAPH 2001, Los Angeles, CA, 12-17 August 2001*, pp. 67-76.

CARR, J.C., BEATSON, R.K., CHERRIE, J.B. MITCHELL, T.J., FRIGHT, W.R., MCCALLUM, B.C. & EVANS, T.R. 2003. Smooth Surface Reconstruction from Noisy Range Data. *ACM GRAPHITE 2003, Melbourne, Australia, 11-19 February 2003*, pp. 119-126.

DONOVAN, J., HANDY, J., KEMENY, J. & O'BRIEN, G. 2005a. Automatic Acquisition and Determination of Rock Discontinuity Properties Using Three-Dimensional Laser Scanning. *Proceedings of Applications of Computers and Operations Research in the Mineral Industry* (APCOM 2005), S. Dessureault Editor, Tucson, AZ.

DONOVAN, J., KEMENY, J., & HANDY, J. 2005b. The Application of Three-Dimensional Imaging to Rock Discontinuity Characterization. *Alaska Rocks 2005: The 40th U.S. Symposium on Rock Mechanics*, Anchorage, AL.

GEOSPATIAL SOLUTIONS. 2004. Scanning the 3D Laser Market. *Geospatial Solutions*, Vol. 14, no. 6. pp. 36-41.

GATH, I. & GEVA, A.B. 1989. Unsupervised Optimal Fuzzy Clustering. *IEEE Trans. Pattern Analysis and Machine Intelligence*, Vol. 11, No 7, pp. 773-781.

HAMMAH, R.E. & CURRAN, J.H. 1998. Fuzzy Cluster Algorithm for the Automatic Identification of Joint Sets. *Int. J. Rock. Mech. Min. Sci.* Vol. 35 (7), pp. 889-905.

JACOBS, G. 2005a. Understanding Laser Scanning Terminology. *Professional Surveyor*, Vol. 25, no. 2, pp. 26-31.

JACOBS, G. 2005b. Laser Scanning: Big Advantages, Even for Small Projects. *Professional Surveyor*, Vol. 25, no. 10, pp. 30-35.

KEMENY, J. & POST, R. 2003. Estimating Three-Dimensional Rock Discontinuity Orientation from Digital Images of Fracture Traces. *Computers and Geosciences* Vol. 29, pp. 65-77.

KEMENY, J., HANDY, J., DONOVAN, J. & THIAM, S. 2004. Automatic Discontinuity Characterization of Rock Faces Using 3D Laser Scanners and Digital Imaging. *Gulf Rocks 2004 (ARMA 2004 Rock Mechanics Symposium and the 6th NARMS)*, Houston, TX.

NASRALLAH, J., MONTE, J. & KEMENY, J. 2004. Rock Mass Characterization for Slope/Catch Bench Design using 3D Laser and Digital Imaging. *Gulf Rocks 2004 (ARMA 2004 Rock Mechanics Symposium and the 6th NARMS)*, Houston, TX.

POBOLINE. 2005. *2005 3D Laser Scanner Hardware & Software Surveys.* http://www.pobonline.com/FILES/HTML/ProductSurveys/

SLOB, S., HACK, R. & TURNER, A.K. 2002. An Approach to Automate Discontinuity Measurements of Rock Faces Using Laser Scanning Techniques. *Proceedings of ISRM International Symposium on Rock Engineering for Mountainous Regions – Eurock 2002,* Funchal, Portugal, pp.87-94.

SLOB, S., HACK, R., VAN KNAPEN, B., TURNER, A.K. & KEMENY, J. 2004. Automated Identification and Characterization of Discontinuity Sets in Outcropping Rock Masses Using 3D Terrestrial Laser Scan Survey Techniques. *Proceedings of Eurock 2004 and 53rd Geomechanics Colloquy*, Salzburg, Austria.

SLOB, S., VAN KNAPEN, B., HACK, R., TURNER, A.K. & KEMENY, J. 2005. A Method for Automated Discontinuity Analysis of Rock Slopes with 3D Laser Scanning. *Transportation Research Record (in press)*, Transportation Research Board, National Academy Press, Washington, D.C.

SPLIT ENGINEERING HOME PAGE. 2005. Split Engineering LLC. http://www.spliteng.com/split-fx/default.asp

TURNER, A.K. 2003. Definition of the Modelling Technologies. *In, New Paradigms in Subsurface Prediction (Rosenbaum & Turner eds), Lecture Notes in Earth Sciences 99,* Springer-Verlag, Berlin, pp. 27-40.

XIE, X.L. & BENI, G.A. 1991. A Validity Measure for Fuzzy Clustering. *IEEE Trans. Pattern Analysis and Machine Intelligence*, Vol. 13 No. 8, pp. 841-847.

ZHOU, W. & MAERZ, N.H. 2002. Implementation of Multivariate Clustering Methods for Characterizing Discontinuities Data from Scanlines and Oriented Boreholes. *Computers and Geosciences*, Vol. 28, pp. 827-839.