

Thresholds of Perception of Repetition Pitch. Conclusions Concerning Coloration in Room Acoustics and Correlation in the Hearing Organ

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Summary

It is shown that KUTTRUFF's "temporal diffusion" Δ , being a measure for the number and the strength of periodic reflections in a room, is a valuable measure too for the perceived coloration of sound ("repetition pitch") in that room caused by such periodicities (comb-filter effect).

The thresholds of perception of repetition pitch may be predicted by means of a short-time autocorrelation model of the hearing organ. A simple relation between the physiological threshold criterion and Δ will prove to be valid.

Wahrnehmungsschwelle der „Wiederholungs-Tonhöhe“. Ergebnisse zur Klangfarbe in der Raumakustik und der Korrelation im Gehör

Zusammenfassung

Es wird gezeigt, daß die von KUTTRUFF eingeführte „zeitliche Diffusität“ Δ , die ein Maß für Zahl und Stärke der periodischen Rückwürfe eines Raumes ist, auch ein gutes Maß für die empfundene Klangfarbe in dem Raum („Wiederholungs-Tonhöhe“) bietet, die durch solche Rückwürfe hervorgerufen wird (Kammfiltereffekt).

Die Wahrnehmungsschwelle der „Wiederholungs-Tonhöhe“ kann mit einem Kurzzeit-autokorrelationsmodell des Gehörs vorausgesagt werden. Man erhält eine einfache Beziehung zwischen dem physiologischen Schwellenkriterium und Δ .

Seuils de perception du ton de répétition. Conclusions concernant la coloration en acoustique de salle et la corrélation avec l'organe d'audition

Sommaire

On montre que la «diffusion temporelle» Δ de KUTTRUFF, qui est la mesure du nombre et de l'intensité des réflexions périodiques dans une salle, est une grandeur précieuse également pour la perception de la coloration du son (ton de répétition) causée dans cette salle par de telles périodicités (effet de filtres en «treillis»).

En utilisant un modèle d'autocorrélation à temps court pour l'organe d'audition, il est possible de prévoir les seuils de perception du ton de répétition.

On prouvera la validité d'une relation simple entre le critère du seuil physiologique et Δ .

1. Introduction

In a room, the original sound (from a source) will be influenced by the transmission characteristics of the room. This implies that, in general, a listener (placed in a certain position) will perceive the original sound as modified or coloured. This coloration may have different origins and is manifest from the frequency or pulse response of the room in principle.

A special kind of coloration occurs when the direct (original) sound is followed by a strong reflection (repetition). This effect is perceived easily under laboratory circumstances, but it may also be observed under certain conditions in studios and concert halls. In this connection we refer to the

findings of KÜHL [1] and SOMERVILLE et al. [2]. The latter reported that in some concert halls with reflectors, speech and music became coloured. They suggested that these "distortions" could be explained as comb-filter effects due to interference between the direct and the reflected sound.

This monaural interaction of a sound with its repetition after a certain delay τ_0 has been studied in more detail by FOURCIN [3] and BILSEN [4] in the field of psychoacoustics. FOURCIN showed that the coloration is due to the existence of a subjective tone with a pitch corresponding to the reciprocal of τ_0 . Phase changes of the delayed sound were studied by the present author; he introduced the name "Repetition Pitch" or in short "RP". (The word "Repetition" indicates the necessary correlation between

original and repeated sound; e.g. RP is absent when the original sound is passed through a high-pass filter and the repetition through a low-pass filter in the absence of a common passband.)

By adding more than one repetition to the original sound, the delay between all subsequent repetitions being equal to a constant τ_0 , the pitch of the tone in question does not change but the timbre does. For example, if the pulse response of a transmission system is that of a flutter echo, white noise at the input will cause a very sharp, metallic timbre (like the sound of a sawing machine or grinding wheel).

Because in almost every room periodic reflections occur, and thus RP may be expected to be perceivable sometimes, it seems worthwhile to look after a quantity which:

- 1) can be measured easily in a room and
- 2) is a proper measure for the perception of RP.

A great part of the present work is based on the work of ATAL, SCHROEDER and KUTTRUFF [5]. They determined the thresholds of perception of coloration in comb-filtered white GAUSSIAN noise, and proposed two models for the explanation of their results. Both models are based on short-time analysis by the hearing organ, one yielding a criterion in the frequency domain, the other in the time domain. In fact their two models are equivalent, differing only in the threshold criteria.

For that reason we checked the validity of both criteria. It will be shown that only the criterion belonging to the short-time autocorrelation model fits the results of our listening tests.

Furthermore, a simple relation will be derived between this subjective threshold criterion and KUTTRUFF's [6] "temporal diffusion" Δ , this latter quantity being a measure for the number and strength of periodic reflections at a certain place in a room.

2. Autocorrelation in room acoustics

2.1. Kuttruff's "temporal diffusion" Δ

The pulse response of a room may be given by

$$h(t) = \sum_k a_k \delta(t - t_k). \quad (1)$$

The autocorrelation function of this response follows [6] as

$$\varphi(\tau) = \delta(\tau) \sum_k a_k^2 + \sum_{k \neq l} a_k a_l \delta(\tau + t_k - t_l). \quad (2)$$

In these equations the instants t_k may be distributed arbitrarily along the time axis (at $t = t_k$ the k -th reflection with strength a_k arrives at the listener's position). Note that the reflections are supposed not to

be distorted by frequency dependent reflection or absorption.

The first term of eq. (2) yields a peak at $\tau = 0$ with a height $\sum a_k^2$. If the instants t_k are distributed completely aperiodically, the second term yields peaks with an average height of $a_k a_l$. In that case the peak at $\tau = 0$ is much higher than all the others. If, however, periodic reflections occur, a certain time interval $t_l - t_k = \tau_0$ will occur frequently; this results in an enhanced peak at $\tau = \tau_0$ too.

Now, KUTTRUFF proposed the ratio Δ of the peak at $\tau = 0$ to the highest peak of the autocorrelogram for any non-zero delay as a quantitative measure for the number and strength of these periodicities; he called this property "zeitliche Diffusität" or "temporal diffusion".

2.2. Δ of an idealized semi-periodic transmission system

In general, RP is due to periodic reflections (repetitions) in a transmission system (like a room, a reverberation spring etc.). Now the question arises: is Δ a proper measure for the perception of RP? To answer this question we have done listening tests starting from an idealized semi-periodic transmission system, which will be described mathematically in this section.

Let us consider a transmission system which possesses periodic and aperiodic reflections. In fact such a system is the superposition of a fully aperiodic and a fully periodic transmission system with period τ_0 . Thus

$$h(t) = h(t)_{\text{aperiodic}} + h(t)_{\text{periodic}} \\ \text{or } h(t) = \sum_m b_m \delta(t - t_m) + \sum_r c_r \delta(t - t_r) \quad (3)$$

with $t_r = r \tau_0$ ($r = 0, 1, 2, 3, \dots; m = 0, 1, 2, 3, \dots$).

The autocorrelation function of this semi-periodic system may be considered to be composed of two autocorrelation functions, one of the aperiodic (first and second term of eq. (4)) and one of the periodic (third and fourth term of eq. (4)) system, and a cross correlation function of these systems (last term of eq. (4)):

$$\varphi(\tau) = \delta\{\tau\} \sum_m b_m^2 + \sum_{m \neq n} b_m b_n \delta\{\tau + t_m - t_n\} + \\ + \delta\{\tau\} \sum_r c_r^2 + \sum_s [\delta\{\tau \pm (s+1)\tau_0\} \sum_r c_r c_{r+s+1}] + \\ + \sum_{m \neq r} b_m c_r \delta\{\tau \pm (t_m - t_r)\}. \quad (4)$$

Now, let us restrict ourselves to the case where the periodic system is a flutter echo having the period τ_0 and an exponential decay with reverberation time T (see Fig. 1). Then

$$c_r = c_0 g^r, \quad \text{with } g = 10^{-3\tau_0/T}. \quad (5)$$

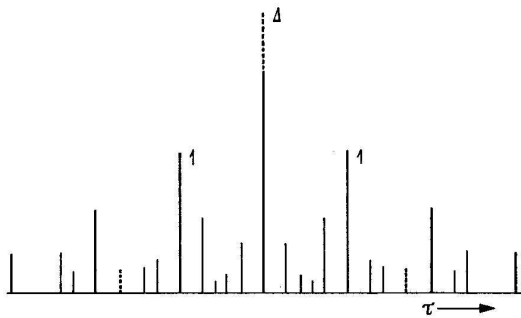


Fig. 1. The autocorrelogram of an ideal semi-periodic system.

- autocorrelation function of the periodic part,
- - - autocorrelation function of the aperiodic part,
- · - crosscorrelation function.

Under these conditions and using the formula for summing geometric series, it may be shown easily that

$$\sum_r c_r c_{r+1} = g \sum_r c_r^2. \quad (6)$$

In this special case Δ takes the form

$$\Delta = \frac{\sum_m b_m^2 + \sum_r c_r^2}{\sum_r c_r c_{r+1}} = \frac{1}{g} \left[1 + \frac{\sum_m b_m^2}{\sum_r c_r^2} \right]. \quad (7)$$

If the signal putted into this idealized semi-periodic system is white noise, $\sum b_m^2$ over $\sum c_r^2$ in eq. (7) represents in fact the ratio of the power of the "aperiodic part" P_{wn} (wn: white noise) to the power of the "periodic part" P_{cn} (cn: coloured noise) of the output signal. Then, we may rewrite eq. (7) as follows

$$\Delta = \frac{1}{g} \left[1 + \frac{P_{wn}}{P_{cn}} \right]. \quad (8)$$

Realizing that there exists hardly any correlation between the aperiodic and the periodic part of the signal and taking into account that white noise transmitted by an aperiodic system is not perceived as coloured, we arrived at the following experiment.

We substituted the aperiodic part of the signal by the white noise from one noise generator and the periodic part by the coloured noise from another (uncorrelated) noise generator, in combination with a comb-filter having the ideal periodic pulse response. By varying the power ratios P_{wn}/P_{cn} a listener will be able to determine the threshold of coloration, and we shall find the relation between this threshold and Δ with the aid of eq. (8).

3. Short-time analysis of comb-filtered white noise by the hearing organ

Now the question arises: how does the hearing organ perceive signals of this kind, consisting of an original sound (like white noise) and one or more repetitions of that sound? Does the ear make an analysis in the frequency domain or in the time domain, or in both?

3.1. Two threshold criteria

ATAL, SCHROEDER and KUTTRUFF [5] proposed two models for the explanation of thresholds of coloration, both based on a short-time analysis. Their starting point was the consideration that the ear cannot make a real FOURIER analysis of a signal, because the future values of the signal are unknown, and the past values cannot possibly be integrated all with the same weight.

Therefore it seems reasonable to modify the FOURIER integral and to define a running spectrum $F(\omega, t)$ with time as a real parameter, like [7]

$$F(\omega, t) \equiv \int_{-\infty}^t f(\vartheta) r(t - \vartheta) e^{-j\omega\vartheta} d\vartheta. \quad (9)$$

Here $r(t)$ is thought to be a physically realizable, real weighting function. If the signal $f(t)$ is continuous like noise, the time average of the running power spectrum, to be referred to as the short-time power spectrum $\Phi_s(\omega)$, is a significant quantity:

$$\Phi_s(\omega) = \overline{|F(\omega, t)|^2} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} F(\omega, t) F^*(\omega, t) dt. \quad (10)$$

This can be shown to be equal to the convolution of $\Phi(\omega)$ and $P(\omega)$, $\Phi(\omega)$ being the power spectrum $F(\omega)F^*(\omega)$ of $f(t)$ and $P(\omega)$ the energy spectrum $R(\omega)R^*(\omega)$ of $r(t)$; thus

$$\Phi_s(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi(\Omega) P(\omega - \Omega) d\Omega. \quad (10 a)$$

ATAL et al. also defined a short-time autocorrelation function $\varphi_s(\tau)$ being the FOURIER transform of the short-time power spectrum. Then using the convolution theorem it follows from eq. (10 a) that

$$\varphi_s(\tau) = \varphi(\tau) \varrho(\tau), \quad (11)$$

where $\varphi(\tau)$ is the autocorrelation function of $f(t)$ and $\varrho(\tau)$ the autocorrelation function of $r(t)$.

For the comb-filters tested by ATAL et al. the two following threshold criteria proved to be valid. In the frequency domain model coloration will be perceived if the level differences between maxima and minima of the short-time power spectrum exceed a

certain threshold A_0 . Thus

$$A_0 = 10 \log \frac{\Phi_s(\omega)_{\max}}{\Phi_s(\omega)_{\min}}. \quad (12)$$

In the time domain (autocorrelation) model coloration will be perceived if the ratio of the maximum value of the short-time autocorrelation function for any non-zero delay to its value at zero delay exceeds a certain threshold B_0 . Thus

$$B_0 = \frac{\varphi_s(\tau)_{\max(\tau \neq 0)}}{\varphi_s(0)}. \quad (13)$$

3.2. Calculation of the autocorrelation function $\varrho(\tau)$, belonging to the weighting function $r(t)$ of the hearing organ, from the results of Atal, Schroeder and Kuttruff

From the results of listening tests with different comb-filters ATAL et al. calculated the weighting function $r(t)$ and its energy spectrum $P(\omega)$ (in their notation: $E(\omega)$), which they presented in graphical form. However, they did not give the autocorrelation function $\varrho(\tau)$ of $r(t)$. As $\varrho(\tau)$ appeared to us to be of practical importance, we reconstructed this function from the test results of ATAL et al. Fortunately, in one simple case, when the original sound is followed by only one repetition (see Fig. 2),

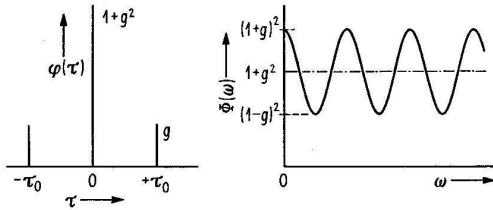


Fig. 2. The autocorrelation function and the power spectrum for a comb-filter due to one repetition with strength g .

the calculation is very simple. In that case eq. (4) takes the form

$$\varphi(\tau) = (1 + g^2)\delta(\tau) + g\delta(\tau \pm \tau_0). \quad (14)$$

Substituting eq. (14) into eq. (11) we find

$$\varphi_s(\tau) = (1 + g^2)\varrho(0)\delta(\tau) + g\varrho(\tau_0)\delta(\tau \pm \tau_0). \quad (15)$$

The FOURIER transforming of eq. (15) yields

$$\Phi_s(\omega) = (1 + g^2)\varrho(0) + 2g\varrho(\tau_0)\cos\omega\tau_0. \quad (16)$$

Finally, by substituting eq. (16) into eq. (12) it follows that

$$A_0 = 10 \log \frac{(1 + g^2)\varrho(0) + 2g\varrho(\tau_0)}{(1 + g^2)\varrho(0) - 2g\varrho(\tau_0)} \quad (17)$$

and by substituting eq. (15) into eq. (13)

$$B_0 = \frac{g\varrho(\tau_0)}{(1 + g^2)\varrho(0)}. \quad (18)$$

As we can easily see, the relation between A_0 and B_0 is a very "direct" one in this particular case, i. e.

$$A_0 = 10 \log \frac{1 + 2B_0}{1 - 2B_0}. \quad (19)$$

In eq. (19) $\varrho(\tau)$ does not occur; this means that, here, the A_0 - and B_0 -criteria are equivalent and, thus, must lead to the same $\varrho(\tau)$.

If $\tau_0 = 0$ ($\varrho(\tau_0) = \varrho(0)$) A_0 and B_0 are derived directly (without knowing $\varrho(\tau)$) from the threshold value of g at $\tau = 0$, being -24 dB ([5], extrapolation in Fig. 2); we find

$$A_0 = 1,1 \text{ dB} \quad \text{and} \quad B_0 = 0,063.$$

Because, apparently, only the ratio $\varrho(\tau_0)/\varrho(0)$ is important (see eqs. (17) and (18)), we may choose for $\varrho(0)$ any constant. Obviously $\varrho(0)$ should be taken equal to 1, because then $\varphi_s(0) = \varphi(0)$. This implies that the ear is capable of correlating the present values of a signal without weighting.

Knowing B_0 and the threshold values of g for the time interval $0 < \tau_0 < 80$ ms ([5], Fig. 2) we calculated $\varrho(\tau)$. The function thus found could be tested and extended for the values $20 < \tau_0 < 160$ ms, using the B_0 -criterion, from the results of tests with other comb-filters ([5], Fig. 3). Calculating A_0 from $\varrho(\tau)$ with eq. (12) yielded values equal to 1.1 ± 0.2 ; indeed, these results are inconclusive with regard to the validity of the A_0 - and B_0 -criterion. A graph of $\varrho(\tau)$ is given in Fig. 3.

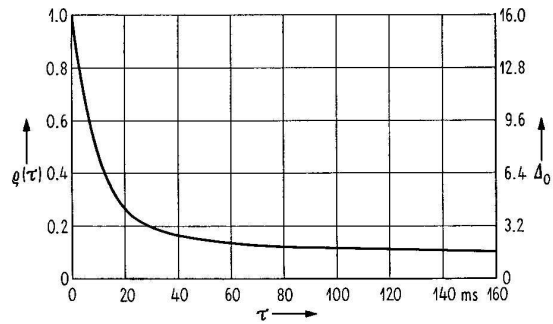


Fig. 3. ϱ and Δ_0 as a function of τ .

4. The experiments

In order to test the A_0 - and B_0 -criterion once again and to determine the relation between Δ and the threshold of RP we undertook the listening tests described fundamentally in Section 2.

4.1. Experimental apparatus

A block diagram of the apparatus is given in Fig. 4. The white noise from noise generator NG 1 (General Radio) is comb-filtered by a reverberation machine (Philips), consisting of an endless tape

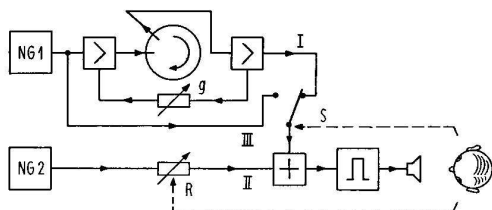


Fig. 4. Block diagram of the experiment.

loop with an erasing and a recording head and eight playback heads. Only two heads are in use, the recording head and one playback head. The signal from the playback head is fed back to the recording head (factor g); thus used, the machine produces a real flutter echo.

The output (I) of the playback head is mixed with the white noise from another, uncorrelated, noise generator NG 2 (General Radio). The sum signal (I + II) is fed through a band-pass filter of 100 Hz to 5600 Hz (Peekel) and an amplifier (omitted in the block diagram) to an electrostatic loudspeaker (Quad) in the anechoic room. The filtering proved to be necessary because of the limited bandwidth of the reverberation machine. After the filtering there was no audible difference between the noise from NG 2 and the noise from NG 1 passed by the reverberation machine without feedback. Moreover, the high frequencies can be omitted because they are of no importance for the perception of RP.

There were two trained subjects. Each subject made two settings of R for each combination of g and τ_0 . In total sixteen different combinations of g and τ_0 have been tested, viz τ_0 being equal to 30, 60, 90 and 120 ms and T being equal to 0.5, 1.0, 1.5 and 2.0 s.

The test signal was presented (two-ear monophonically) at a sound pressure level of about 65 dB (re $2 \cdot 10^{-5} \text{ Nm}^{-2}$).

After each setting of R the experimenter measured the critical level difference $L_{wn} - L_{cn}$ ($= L_{II} - L_I$).

4.3. Results

From our test results we calculated the critical values of A , to be noted as A_0 , with the aid of eq. (8). According to the definitions of A and B_0 (eq. (13) with eq. (11)) these values must satisfy the relation

$$B_0 = \frac{\varrho(\tau_0)}{A_0}. \quad (20)$$

Substituting in this eq. (20) the values of A_0 with the corresponding values of $\varrho(\tau_0)$ from Fig. 3 and averaging these so found values of B_0 we get $B_0 = 0.059$, which is in good agreement with $B_0 = 0.063$ from the results of ATAL et al.

In order to compare the A_0 -criterion with the B_0 -criterion we also calculated A_0 with the aid of the following equation

$$A_0 = 10 \log \frac{\left[1 + \frac{P_{wn}}{P_{cn}} \right] + 2g\varrho(30) + 2g^2\varrho(60) + 2g^3\varrho(90) + 2\frac{g^4}{1-g}\bar{\varrho}}{\left[1 + \frac{P_{wn}}{P_{cn}} \right] - 2g\varrho(30) + 2g^2\varrho(60) - 2g^3\varrho(90) + 2\frac{g^4}{1+g}\bar{\varrho}}. \quad (21)$$

To enable the subject to make a direct comparison between the semi-coloured white noise (I + II) and white noise at the same sound pressure level, the signal of NG 1 is also fed directly (III) to the mixing amplifier. The level of signal I is adjusted equal to the level of signal III by the experimenter. Thus, I + II and III + II are on the same level too.

The apparatus to measure the reverberation time T of the flutter echo is not shown in the block diagram; g is found from T by eq. (5).

4.2. Task of the subjects

The subject could compare the signal I + II with the signal III + II by means of the switch S. He was asked to increase the level II, handling the potentiometer R, until he could no longer distinguish the transition from I to III and vice versa. The threshold of RP, thus found, is a "threshold from above".

Eq. (21) was derived from the general expression eq. (4) in the same manner as eq. (17). Moreover, use was made of eqs. (5) and (8), and τ_0 was introduced at 30 ms. The last term in the nominator of eq. (21) is the sum of a geometric series of rest terms, multiplied by $\bar{\varrho}$, this being the assumed average of the (partly unknown) values of $\varrho(\tau)$ for $\tau \geq 120$ ms; we took $\bar{\varrho} = 0.10$ (see Fig. 3). The last term in the denominator is the sum of an alternating geometric series of the same rest terms.

Neglecting the third and subsequent terms in the nominator and denominator of eq. (21) we find an A_0' which in fact obeys the B_0 -criterion, the relation between A_0' and B_0 being the analytic one of eq. (19). The values of A_0 and A_0' , calculated in this way from our test results, are given in Fig. 5. Note that there is full agreement between the values of A_0' and those found by ATAL et al. for A_0 , viz $A_0' = (1.1 \pm 0.2)$ dB. On the contrary, our values

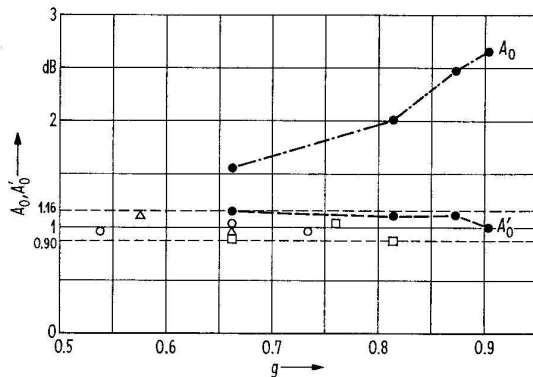


Fig. 5. Calculated values of A_0 and A'_0 .

- $\tau_0 = 30$ ms,
- $\tau_0 = 60$ ms,
- $\tau_0 = 90$ ms,
- △ $\tau_0 = 120$ ms.

of A_0 show an unpermissible deviation from this value. We may therefore conclude that the B_0 -criterion is to be preferred.

5. Discussion

It is not surprising that ATAL et al. did not find a significant deviation in the values of A_0 , because at high values of g there were only few higher order terms of any magnitude in the calculation ([5], comb-filter 5). Or, in those cases where higher order terms of importance were to be found in the calculation, g proved to be small and consequently the higher order terms did not have much influence ([5], comb-filter 6).

Summarising, we are inclined to conclude that the threshold of perception of RP may be explained by short-time autocorrelation in the hearing organ, not by short-time frequency analysis.

Secondly, as the B_0 -criterion proved to be valid for two extreme cases of periodicity in room acoustics, viz 1) the combination of direct sound with one strong reflection (e.g. due to a reflector above the sound source) and 2) the ideal flutter echo, we tend to accept it as a general criterion for the perception of RP in room acoustics.

The B_0 -criterion is based on listening tests with white noise as input signal. For speech and music greater periodicities are expected to be tolerable. In this regard the experiments of SOMERVILLE et al. [2]

are quite informing. He found the threshold (from above) of perceptibility of a 10 ms-echo to be equal to -16.0 dB for random noise (with constant energy per octave band) and -12.1 dB for speech, etc. The value -16 dB for noise is in agreement with the corresponding value of ATAL et al. The threshold difference for noise and speech, being thus only 4 dB, we conclude that the B_0 -criterion is not too severe a criterion, but a safe one. For practical purposes we added a special scale to Fig. 3, calculated with the aid of eq. (20), which enables us to read directly from this figure, for known values of Δ and τ_0 , if Repetition Pitch will be perceived or not.

Acknowledgements

I would like to thank Prof. Dr. Ir. C. W. KOSTEN for reading the manuscript.

I am grateful to Ir. D. W. VAN WULFFTEN PALTHE for fruitful discussions.

Special thanks are given to Dr. H. KUTTRUFF from University of Göttingen for his advice regarding the reconstruction of $\varrho(\tau)$ from article [5].

I am indebted to Mr. W. M. SCHULLER for his help during the listening tests.

(Received October 14th, 1966.)

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