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Testing cometary ejection models to fit the 1999 Leonids and to predict future showers

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ABSTRACT

Brown & Jones have discussed four models for the ejection velocities of cometary material during the perihelion passage of a comet. The ejection velocities depend on the heliocentric distance, density and mass of the ejected particle and on the size of the comet. For the density of the particles, they assumed three values, 100, 800 and 4000 kg m^{-3}, which give a possible combination of 12 cometary ejection models. This paper tests whether these models can be applied to explain the Leonid phenomenon for which Comet 55P/Tempel–Tuttle is responsible. The ejection is simulated of 61 000 particles during each of the last four apparitions of Tempel–Tuttle, and flux rates on Earth are derived. The results prove that the models with a particle density of 4000 kg m^{-3} provide the best fit to the observed Leonid rates in 1999. Based on the ejection velocity model that gives the smallest root-mean-square error for the year 1999, noticeable displays of the Leonids are predicted for 2000 and 2001 November.

Key words: comets: individual: 55P/Tempel–Tuttle – meteors, meteoroids – Solar system: general.

1 INTRODUCTION

The Leonids are small dust particles which enter the upper atmosphere of the Earth with a very high velocity of about 71 km s^{-1}. They are the result of the passage of the Earth through dust trails of Comet 55P/Tempel–Tuttle. The meteor activity associated with Comet Tempel–Tuttle is called a 'Leonid' event because the meteors appear to be coming from the direction of the constellation Leo. There is evidence that this comet has created meteor showers and meteor storms for more than 1000 years. Tempel–Tuttle, named after Ernst Tempel and Horace Tuttle who first discovered the comet in 1865 and 1866, has a nuclear radius of about 2 km (Hainaut et al. 1998) and orbits the Sun with a period of just over 33 yr. At its perihelion it passes close to the orbit of the Earth. The last perihelion passage occurred on 1998 February 28.

Until recently the capabilities of predicting the time and the intensity of the Leonids were quite poor. However, new research studies, especially those performed by Kondrat'eva & Reznikov (1985), Kondrat'eva, Murav'eva & Reznikov (1997), McNaught & Asher (1999) and Brown (2000), have provided a better understanding of the phenomenon of the Leonids. McNaught & Asher claim that the time of maximum is now predictable to 10-min accuracy or better.

In this paper the cometary ejection models of Brown & Jones (1998) are applied in order to simulate the dust population of Tempel–Tuttle released since 1866 (Section 2). The dust particles are propagated with a Stumpff–Weiss orbit propagator (Section 3), and encounters with the Earth in the year 1999 are analysed for all models (Section 4). Finally in Section 5, predictions for the years 2000 and 2001 are made, based on the model, that best fit the observational data on the 1999 Leonids.

2 COMETARY EJECTION MODELS

When a comet approaches the Sun, sublimation of volatiles (primarily water ice) sets in, and particles are caused by momentum transfer to leave the parent comet. Reflecting the considerable uncertainty about the ejection process, four different models have been developed by Brown & Jones (1998) to describe the initial formation of the meteoroid stream. They are based on Whipple's (1951) 'icy snowball' model:

$$v_{\mathrm{eject}} = 8.03 r^{-1.125} \rho^{-1/3} r_{\mathrm{c}}^{1/2} m^{-1/6} f,$$

where r is the heliocentric distance in au, ρ and m are the density and mass of the grain, respectively, r_{c} is the radius of the cometary nucleus and f is the fraction of incident solar radiation used in sublimation; the latter is set to 1 throughout.

The Whipple formula was slightly changed by (among others) Jones (1995) to correct the assumption of a blackbody-limited nucleus temperature as well as the neglect of the adiabatic expansion of the gas. This corrected formula is chosen as the first model:

$$v_{\mathrm{eject}} = 10.2 r^{-1.038} \rho^{-1/3} r_{\mathrm{c}}^{1/2} m^{-1/6}.$$

To allow, furthermore, grains of the same mass but with

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Table 1. The formulae for the ejection velocity according to the four model variants. The units are $(r_{\mathrm{grain}}) = \mathrm{cm}$, $(r) = \mathrm{au}$, $(r_{\mathrm{c}}) = \mathrm{km}$, $(\rho) = \mathrm{g\,cm^{-3}}$ and $(m) = \mathrm{g}$ to get the ejection velocity in $\mathrm{m\,s^{-1}}$ (Brown & Jones 1998).

Model no.	Name	Ejection formula
1	Jones ejection distribution	$v_{\mathrm{eject}} = 10.2r^{-1.038}\rho^{-1/3}r_{\mathrm{c}}^{1/2}m^{-1/6}$
2	Jones ejection distribution with parabolic probability distribution	$v_{\mathrm{eject}} = 10.2r^{-1.038}\rho^{-1/3}r_{\mathrm{c}}^{1/2}m^{-1/6}$ $P(v - v_{\mathrm{eject}}) = 1 - \left(\frac{v}{v_{\mathrm{eject}}} - 1\right)^2$ for $0 < v < 2v_{\mathrm{eject}}$
3	Jones ejection distribution with modified heliocentric velocity dependence	$v_{\mathrm{eject}} = 10.2r^{-0.5}\rho^{-1/3}r_{\mathrm{c}}^{1/2}m^{-1/6}$
4	Crifo distributed production	$\log_{10}(v_{\mathrm{eject}} + v_0) = -2.143 - 0.605\log_{10}(r_{\mathrm{grain}}) - 0.5\log_{10}r$ with $v_0^2 = 2\mu/r_{\mathrm{c}}$ and $\mu = GM$ $P(v - v_{\mathrm{eject}}) = \frac{1}{\mathrm{e}^{3.7}}\exp\left[\frac{3.7-10.26(v-v_{\mathrm{eject}})+4.12(v-v_{\mathrm{eject}})^2}{1-1.03(v-v_{\mathrm{eject}})+0.296(v-v_{\mathrm{eject}})^2}\right]$

different shapes to reach different velocities, a parabolic distribution is added for the second model:

$$P(v - v_{\mathrm{eject}}) = 1 - \left(\frac{v}{v_{\mathrm{eject}}} - 1\right)^2$$

in the range of $0 < v < 2v_{\mathrm{eject}}$.

The Whipple-derived theories, as do many others, suggest a variation of ejection velocity with heliocentric distance that is of the form $v_{\mathrm{eject}} \propto r^n$, where in the above cases n is close to -1. Since from observations of coma ejection/halo expansions a value of -0.5 can also be deduced, this possibility is adopted in the third model:

$$v_{\mathrm{eject}} = 10.2r^{-0.5}\rho^{-1/3}r_{\mathrm{c}}^{1/2}m^{-1/6}.$$

For the fourth model a different approach is used. Besides the surface as the source of meteoroid grains, sublimation and meteoroid production are now considered to occur throughout the coma. This concept of 'distributed' production in the coma was investigated by Crifo (1995) in his physicochemical model of the inner coma. The ejection process can thus be expressed by the formula

$$\log_{10} v = -2.143 - 0.605\log_{10}(r_{\mathrm{grain}}) - 0.5\log_{10}r,$$

where r_{grain} is the radius of the meteoroid grain. Furthermore, the escape velocity of the particle from the comet has to be subtracted to obtain the final ejection velocity, which is then used together with Crifo's distribution for the differential flux:

$$P(v - v_{\mathrm{eject}}) = \frac{1}{\mathrm{e}^{3.7}}\exp\left[\frac{3.7 - 10.26(v - v_{\mathrm{eject}}) + 4.12(v - v_{\mathrm{eject}})^2}{1 - 1.03(v - v_{\mathrm{eject}}) + 0.296(v - v_{\mathrm{eject}})^2}\right].$$

A summary of the employed ejection models is given in Table 1.

As can be seen in the above-presented formulae, the ejection velocity varies as a function of the density and mass of the grains. To complete the modelling of the ejection process, it is thus necessary to have a closer look at these parameters. Estimates regarding the density of meteoroids are widespread and vary between 100 and 4000 $\mathrm{kg\,m^{-3}}$. To cover this range, Brown & Jones (1998) used three distinct values of 100, 800 and 4000 $\mathrm{kg\,m^{-3}}$. This means that, instead of the original four models, 12 model variants are now examined. The density is denoted by a second number appended to the model number: for example, models 11, 12 and 13 denote the first model with meteoroid densities of 100, 800 and 4000 $\mathrm{kg\,m^{-3}}$, respectively.

To simulate the ejection of dust grains of different sizes, the masses of the meteoroids are varied between 10^{-5} and 10 g. Over this range, 61 mass categories are used, each being 0.1 greater in $\log m$ than the previous one. In each mass category the ejection of 1000 meteoroids is simulated (assuming a logarithmic distribution in the absence of a better mass function), totalling 61 000 particles per model and per perihelion passage.

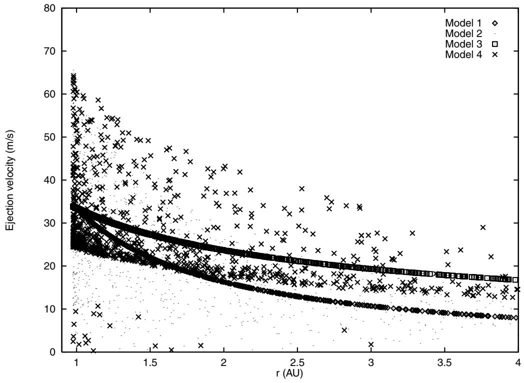

Figure 1. Ejection velocity distributions as a function of heliocentric distance according to the four models. For each model, 1000 meteoroids were simulated with a mass of 0.01 g and a density of 800 $\mathrm{kg\,m^{-3}}$.

For the ejection activity a limit of 4 au is chosen, accounting for more volatile compounds than water, since water sublimation fails at distances of more than 3 au. Hence the ejection process is considered to start when the comet approaches the Sun to a distance of less than 4 au. The number of ejected particles depends upon the amount of incoming solar energy and is thus proportional to r^{-2}. On the other hand, from Kepler's second law it is evident that the probability for the comet to be at a heliocentric distance r is proportional to r^2. This implies that the number of ejected particles is uniformly distributed for all values of true anomaly within the active segment of the cometary orbit.

Additionally, sublimation occurs only on the sunward side of the cometary surface, and the activity reaches its maximum in the direction of the Sun, whereas the ejection process decreases towards the edge of the sunlit area. This is modelled by a $\cos\alpha$ factor, where the direction of the ejected particle is described by its angle α to the comet–Sun line.

In Fig. 1 the ejection velocity according to the four models is plotted as function of the heliocentric distance. For this plot, 1000 meteoroids with a density of 800 $\mathrm{kg\,m^{-3}}$ and a mass of 0.01 g were simulated.

3 THE NUMERICAL INTEGRATOR

In order to integrate thousands of particles over more than 100 yr, a fast but still reliable orbit propagator is needed. Therefore a

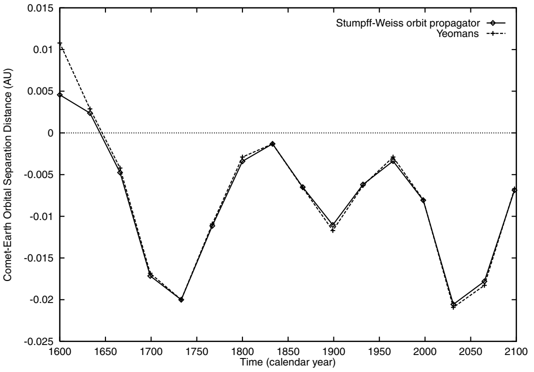

Figure 2. Minimum distances between Comet Tempel–Tuttle and the orbit of the Earth at the time of the comet's passage through its descending node. The Stumpff–Weiss orbit propagator gives identical results to those calculated by Yeomans (1998).

Stumpff–Weiss orbit generator was chosen (Stumpff & Weiss 1968). In the Stumpff–Weiss method the solution of the *N*-body problem is computed as a linear combination of elementary Keplerian orbits for all the possible couples of bodies, providing a fourth-order approximation of state and a third-order approximation of velocity, although higher orders are attainable by integrating the acceleration remainders. For this method, the formulation is symmetrical with respect to all the bodies irrespective of their gravitational influence and, consequently, the concepts of sphere of influence and of central body are no longer used and it is not necessary to reformulate the equations during the integration.

The Stumpff–Weiss orbit generator has been used to propagate Comet Tempel–Tuttle backward in time, taking into account the gravitational perturbations caused by Earth, Mars, Jupiter, Saturn and Uranus. The major perturbations are due to Jupiter. Fig. 2 shows the minimum distances between the comet and the orbit of Earth at the time of passage of the comet through its descending node. The results are identical to those of Yeomans (1998). Close approaches can be seen in the years 1833 and 1966. In both years the Leonids gave a spectacular display.

The effects of solar radiation pressure, which are increasingly dominant with decreasing particle size, are taken into account by adjusting the position and the velocity of the particle after each integration step (Prieto-Llanos 1985). Tests with a Runge–Kutta integrator of order 7/8 have revealed that the inclusion of solar radiation pressure introduces a systematic bias. Göckel (2000), therefore, has introduced a size-dependent correction factor which reduces the effect of the solar radiation pressure and reduces the difference between the two orbit propagators to an acceptable level.

4 BEST MODEL TO FIT THE OBSERVATIONS OF 1999

After intensive testing of the orbit propagator, the orbits of the ejected particles have been calculated. First the comet is integrated backward in time, so that meteoroids can be ejected during the last perihelion passages of the comet. The last perihelion passage occurred in 1998, and every 33 yr before that. 1866 has been chosen as the beginning of our simulations, since the results of Kondrat'eva et al. (1997), McNaught & Asher (1999) and Brown (2000) have shown that previous returns did not actually contribute to the 1999 Leonids. Thus 61 000 meteoroids per model have been ejected around the years 1866, 1899, 1932 and 1965 under the conditions described in Section 2.

Table 2. Measured Leonid zenithal hourly rate of 1999 (Gyssens 1999).

Sol. Long.	Time (UT)	Rate (ZHR)
234.344	Nov 17, $3^{\underline{39}}$	14
234.527	$8^{\underline{00}}$	16
234.951	$18^{\underline{05}}$	30
235.052	$20^{\underline{30}}$	53
235.178	$23^{\underline{30}}$	82
235.217	Nov 18, $0^{\underline{26}}$	210
235.233	$0^{\underline{48}}$	370
235.248	$1^{\underline{10}}$	560
235.263	$1^{\underline{32}}$	1160
235.275	$1^{\underline{48}}$	2360
235.278	$1^{\underline{53}}$	3430
235.282	$1^{\underline{58}}$	2820
235.286	$2^{\underline{04}}$	5400
235.289	$2^{\underline{09}}$	3540
235.298	$2^{\underline{22}}$	2110
235.310	$2^{\underline{38}}$	1140
235.323	$2^{\underline{57}}$	690
235.353	$3^{\underline{40}}$	240
235.383	$4^{\underline{23}}$	153
235.435	$5^{\underline{37}}$	57
235.490	$6^{\underline{56}}$	62
235.532	$7^{\underline{56}}$	51
235.568	$8^{\underline{47}}$	57
235.618	$9^{\underline{58}}$	59
235.741	$12^{\underline{54}}$	56
235.808	$14^{\underline{30}}$	90
235.973	$18^{\underline{25}}$	106
236.338	Nov 19, $3^{\underline{06}}$	23

Since only very few simulated meteoroids would actually hit the Earth, and constraints arising from computer power limit their total number, it has been found necessary to adopt a temporal and spatial 'smearing' to get useful results. This means that particles passing the Earth at a small distance are also regarded as hits. Regarding the choice of this maximum miss distance, the reader is referred to Brown (2000) who suggested a spatial deviation of 0.001 au and a temporal deviation of 0.02 yr (about 1 week) to be appropriate. The same spatial deviation has been adopted, but the temporal deviation has been reduced to 0.01 yr. Hence a particle with a descending node – which marks its minimal distance to the Earth – that lies within 0.001 au of the orbit of the Earth is counted as a hit, provided that the Earth passes the descending node of the particle within half a week before or after the particle itself reached this point. The closer the particle passes to the Earth, the higher weight it has in the simulated flux rate. The weight function is

$$f(x, y) = (0.001 - x)(3.6524 - y),$$

where x is the spatial and y the temporal distance between the particle and the Earth in units of au and days, respectively. Furthermore, a triangular function is attributed to each impacting particle, where the height of the triangle is determined by the weight of the hit described above, and the width of its base is fixed to $0\overset{\circ}{.}05$ of solar longitude. The latter causes the location of the hit to be smeared out over a duration of 1.2 h – which is what it takes the Earth to pass $0\overset{\circ}{.}05$ of solar longitude. All hits are summed over the time or solar longitude.

The predicted meteor rates have to be compared with the

Table 3. Ordering of the models according to the root-mean-square residual between simulated and measured Leonid rates of 1999. The first number of the model variant indicates the ejection velocity model as described in Table 1, and the second number indicates the particle density (1: 100 kg m^{-3}, 2: 800 kg m^{-3}, 3: 4000 kg m^{-3}).

	Model	RMS residual ($\times 10^8$)
1	43	1.851 9111
2	13	3.256 2976
3	33	3.341 1541
4	22	4.077 2193
5	42	4.161 1399
6	23	4.344 4374
7	32	4.634 4712
8	12	4.714 4194
9	31	4.716 2975
10	11	5.362 8643
11	41	5.843 8300
12	21	5.852 9747

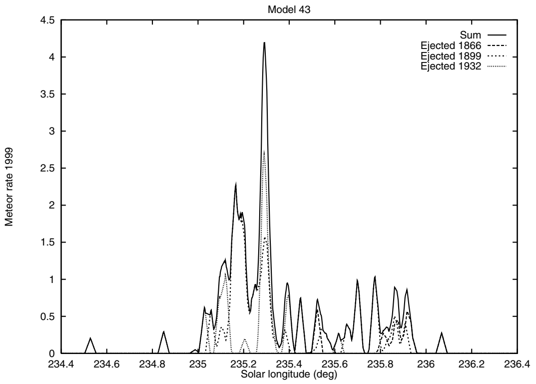

Figure 3. The simulated Leonid rate in the year 1999 for model 43. Note that the measured Leonid peak of 1999 (Table 2) occurred at a solar longitude of 235°.286. The position of the maximum of the above curve coincides perfectly with this value.

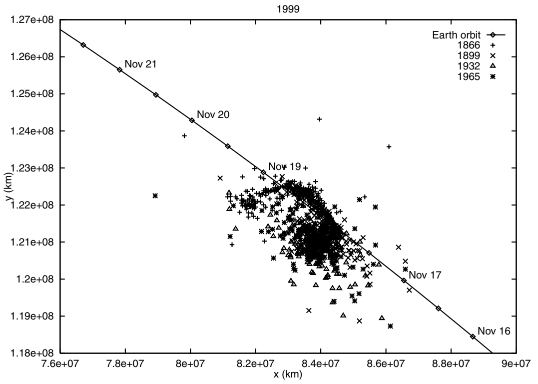

Figure 4. The descending nodes of particles crossing the ecliptic plane in the year 1999 one week before or after the Earth passed the point of minimum distance (as simulated with model 43). The ejection years of the particles are indicated by different symbols, and thus it becomes clear which ejection periods were mainly responsible for the Leonid storm in 1999. The line marks the orbit of the Earth with labels indicating the date (midnight) when the Earth reached the corresponding position.

measured Leonid rate of 1999 (Table 2), so that a ranking of the models can be made. For this purpose the 12 simulated curves are first scaled by a factor of 500 so that they match the order of magnitude of the measured Leonid zenithal hourly rate, and then the method of least squares is applied. This yields the numbers shown in Table 3, where the models are ordered by their root-mean-square residual between the simulated and measured Leonid rates of 1999.

The best fit to the observed data is achieved with model 43. As can also be seen in Table 3, the models with a higher density of the meteoroids (end-number ‘3’) fit better than those with lower densities. The models assuming a particle density of 100 kg m^{-3} (end-number ‘1’) provide the worst fit. This means that the ejection velocity distribution has less influence on the evolution of the dust trails than does the particle density which dictates the effects of the solar radiation pressure. This is confirmed by Lyytinen’s simulations (1999) which show evolution of the dust trails that is very similar to our results, although extremely low ejection velocities are assumed (1 m s^{-1} and lower).

In Fig. 3 the Leonid rate is presented for the year 1999 as simulated with model 43. A closer look reveals that the peak in the meteor rate is caused by particles that were ejected in the periods of 1899 and 1932, whereas the secondary peaks around a solar longitude of 235°.8 are due to particles ejected in 1866. This is in agreement with the other models, too. In Fig. 4 the dust clouds of particles ejected in different periods are plotted for the time when they cross the ecliptic plane in 1999 mid-November, based on model 43. The positions of the dust clouds with respect to each other can be seen. In 1999 the Earth is going directly through the clouds of 1899 and 1932 and – half a day later – through the dust cloud ejected in 1866. This explains the spectacular high Leonid peak observed in 1999. These results are in good agreement with the results of McNaught & Asher (1999). A nice graphical presentation showing the trajectory of the Earth passing through (or by) the particle trails of the individual ejection periods is given by Asher (2000). It can also be found on the website of Armagh Observatory at http://www.arm.ac.uk/leonid/dustexpl.html.

5 PREDICTIONS FOR THE YEARS 2000 AND 2001

Since the models with a high particle density give the best fit of the observed Leonid rates in the year 1999, they are applied to predict the date, time and activity level of the Leonid showers in the years 2000 and 2001. The maximum in the year 2000 is expected to take place at a solar longitude around 235°.27. Models 13, 33 and 43 predict the peak to be between 07:30 and 08:20 UT on 2000 November 17 with a secondary peak 24 h later. The activity level for model 43 is nearly half as high as in 1999, as can be seen in Fig. 5. Fig. 6 reveals that this peak at a solar longitude of 235°.27 is caused by particles ejected in 1932, whereas the secondary peak is caused by ‘1866 particles’. The particles ejected in 1899 which were mainly responsible for the storm in 1999 are no longer dominant. This is in perfect agreement with the results from Kondrat’eva et al. (1997), Lyytinen (1999) and McNaught & Asher (1999), who included ejection of cometary material even back to the year 1733. According to their simulations, the Earth will pass through the cloud of ‘1733 particles’ in the early morning hours of 2000 November 18.

According to our simulations, the activity will further decrease in the year 2001. Fig. 7 shows that the activity may reach less than

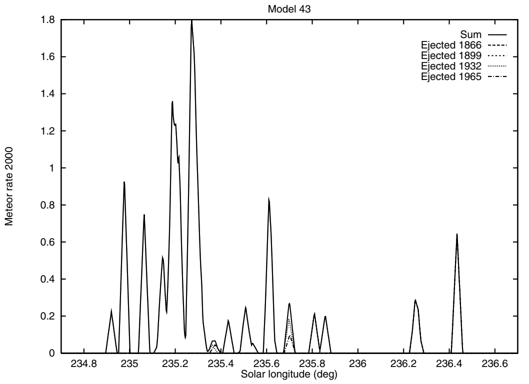

Figure 5. The simulated Leonid rate in the year 2000 for model 43. (The scale of the *y*-axis is arbitrary. If it is multiplied by 500, it is of the order of magnitude of the expected zenithal hourly rates.)

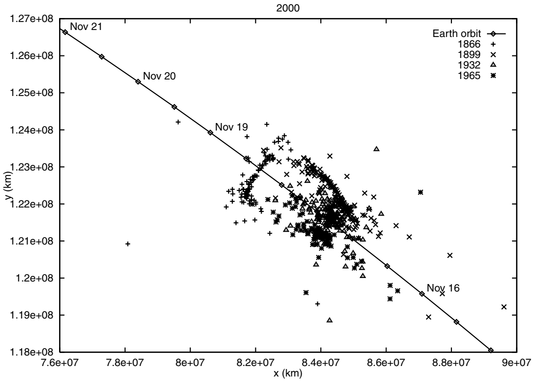

Figure 6. The descending nodes of particles crossing the ecliptic plane in the year 2000 one week before or after the Earth passed the point of minimum distance (as simulated with model 43). The ejection years of the particles are indicated by different symbols. The line and the labels indicate the position of the Earth.

one-third of the 1999 level, which would nevertheless mean a considerable Leonid shower. The first major activity is expected for 2001 November 17 after 12:00 UT,with a second peak (which in model 43 exceeds the first peaks) in the evening (UT) of 2001 November 18. Fig. 8 shows that the '1899 particles' now have almost no contribution. The first peaks are caused by the '1932 and 1965 particles', whereas the '1866 particles' are predicted to create the major peak on November 18. Again these results agree well with those from McNaught & Asher (1999). However, they also predict another major episode of Leonid activity on November 18 at 10:00 UT, arising from particles ejected around the year 1767 which have not been simulated in this paper.

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REFERENCES

Asher D. J., 2000, in Arlt R., ed., Proc. Int. Meteor Conference. International Meteor Organization, in press

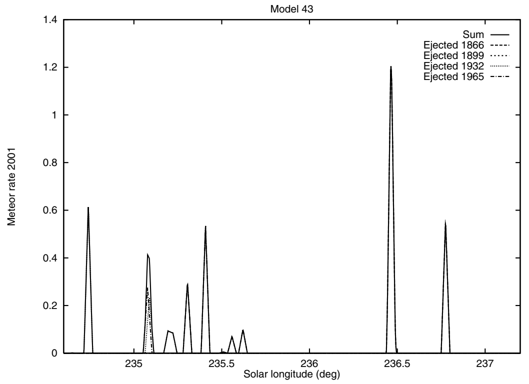

Figure 7. The simulated Leonid rate in the year 2001 for model 43. (The scale of the *y*-axis is arbitrary. If it is multiplied by 500 it is of the order of magnitude of the expected zenithal hourly rates.)

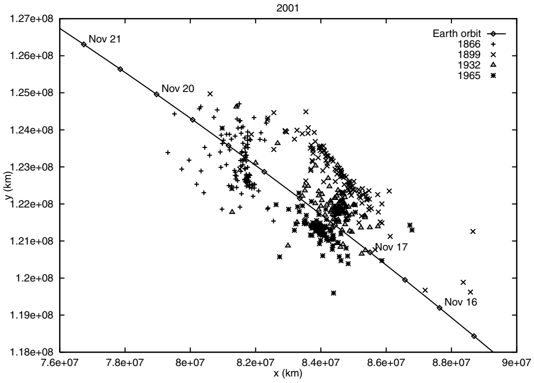

Figure 8. The descending nodes of particles crossing the ecliptic plane in the year 2001 one week before or after the Earth passed the point of minimum distance (as simulated with model 43). The ejection years of the particles are indicated by different symbols. The line and the labels indicate the position of the Earth.

Brown P., 2000, Leonids 1998 Tracking Campaign & Data Analysis, ESA/ESOC Study Report
Brown P., Jones J., 1998, Icarus, 133, 36
Crifo J. F., 1995, ApJ, 445, 470
Göckel C., 2000, Diplomarbeit, TU Darmstadt
Gyssens M., 1999, IMO-News mailing list, 1999 November 20, available by FTP at ftp://ftp.imo.net/pub/imo-news/
Hainaut O. R., Meech K. J., Boenhardt H., West R. M., 1998, A&A, 333, 746
Jones J., 1995, MNRAS, 275, 773
Kondrat'eva E. D., Reznikov E. A., 1985, Sol. Sys. Res., 19, 96
Kondrat'eva E. D., Murav'eva I. N., Reznikov E. A., 1997, Sol. Sys. Res., 31, 489
Lyytinen E., 1999, Meta Res. Bull., 8, 33
McNaught R. H., Asher D. J., 1999, WGN, 27, 85
Prieto-Llanos T., 1985, SWORGE User's Guide, MASLIB Note No. 2, Mission Analysis Office. ESOC, Darmstadt, Germany
Stumpff K., Weiss E. H., 1968, J. Astron. Sci., 15, 5, 257
Whipple F. L., 1951, AJ, 113, 464
Yeomans D. K., 1998, LEONID Meteoroid Storm and Satellite Threat Conference, Manhattan Beach, CA

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